

Pre-AP Geometry Date _____ 4.3 & 4.4 Notes

Proving Triangles Congruent (pp 212–215, 220–222)

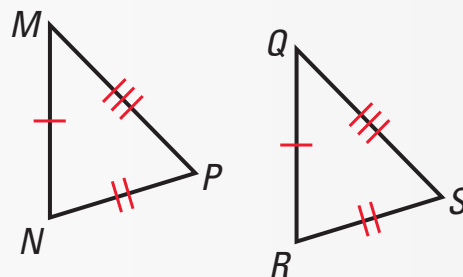
- I can state the ways to prove triangles are congruent.
- I can prove that triangles are congruent.
- I can state the ways that won't work for to prove triangles are congruent.

POSTULATE

POSTULATE 19 *Side-Side-Side (SSS) Congruence Postulate*

If three sides of one triangle are congruent to three sides of a second triangle, then the two triangles are congruent.

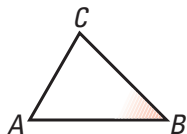
If **Side** $\overline{MN} \cong \overline{QR}$,
 Side $\overline{NP} \cong \overline{RS}$, and
 Side $\overline{PM} \cong \overline{SQ}$,
 then $\triangle MNP \cong \triangle QRS$.



STUDENT HELP

Study Tip

In the triangle, $\angle B$ is the **included angle** between sides \overline{AB} and \overline{BC} .

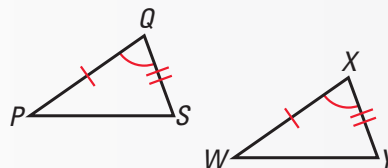


POSTULATE

POSTULATE 20 *Side-Angle-Side (SAS) Congruence Postulate*

If two sides and the included angle of one triangle are congruent to two sides and the included angle of a second triangle, then the two triangles are congruent.

If **Side** $\overline{PQ} \cong \overline{WX}$,
 Angle $\angle Q \cong \angle X$, and
 Side $\overline{QS} \cong \overline{XY}$,
 then $\triangle PQS \cong \triangle WXY$.

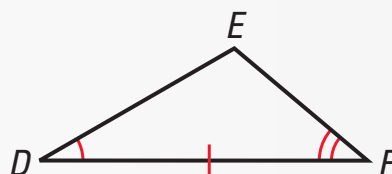
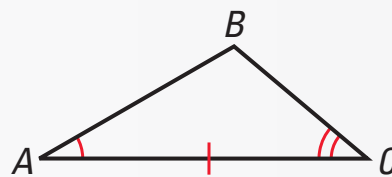


MORE WAYS TO PROVE TRIANGLES ARE CONGRUENT

POSTULATE 21 Angle-Side-Angle (ASA) Congruence Postulate

If two angles and the included side of one triangle are congruent to two angles and the included side of a second triangle, then the two triangles are congruent.

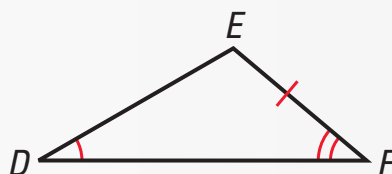
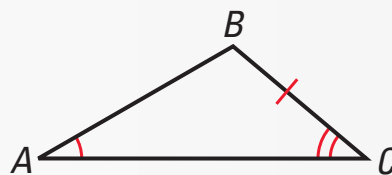
If **Angle** $\angle A \cong \angle D$,
Side $\overline{AC} \cong \overline{DF}$, and
Angle $\angle C \cong \angle F$,
 then $\triangle ABC \cong \triangle DEF$.



THEOREM 4.5 Angle-Angle-Side (AAS) Congruence Theorem

If two angles and a nonincluded side of one triangle are congruent to two angles and the corresponding nonincluded side of a second triangle, then the two triangles are congruent.

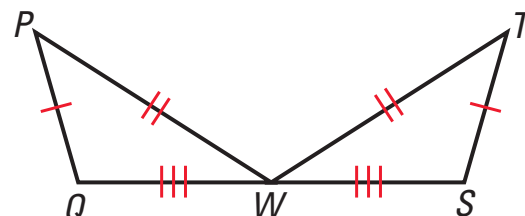
If **Angle** $\angle A \cong \angle D$,
Angle $\angle C \cong \angle F$, and
Side $\overline{BC} \cong \overline{EF}$,
 then $\triangle ABC \cong \triangle DEF$.



EXAMPLE 1 Using the SSS Congruence Postulate

Prove that $\triangle PQW \cong \triangle TSW$.

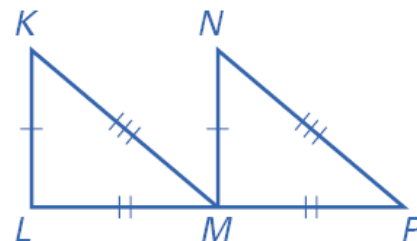
Paragraph Proof The marks on the diagram show that $\overline{PQ} \cong \overline{TS}$, $\overline{PW} \cong \overline{TW}$, and $\overline{QW} \cong \overline{SW}$.



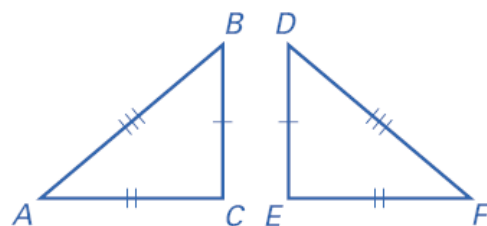
► So, by the SSS Congruence Postulate, you know that $\triangle PQW \cong \triangle TSW$.

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1. Prove $\triangle KLM \cong \triangle NMP$

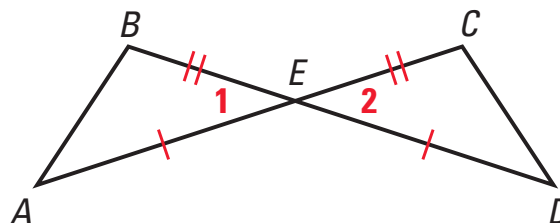


2. Prove $\triangle ABC \cong \triangle FDE$



EXAMPLE 2 **Using the SAS Congruence Postulate**

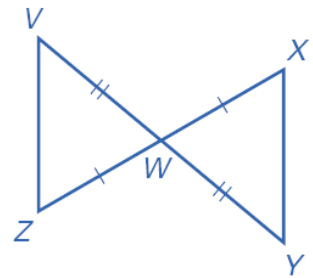
Prove that $\triangle AEB \cong \triangle DEC$.



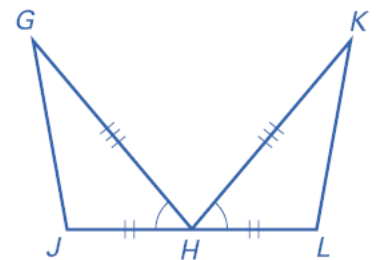
Statements	Reasons
1. $\overline{AE} \cong \overline{DE}, \overline{BE} \cong \overline{CE}$	1. Given
2. $\angle 1 \cong \angle 2$	2. Vertical Angles Theorem
3. $\triangle AEB \cong \triangle DEC$	3. SAS Congruence Postulate

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3. Prove: $\triangle VWZ \cong \triangle YWX$

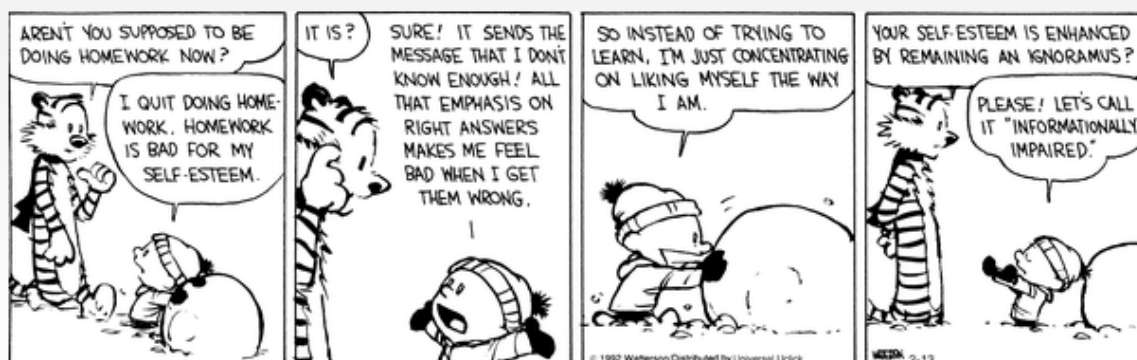


4. Prove $\triangle GHJ \cong \triangle KHL$



Calvin and Hobbes by Bill Watterson

February 13, 2012

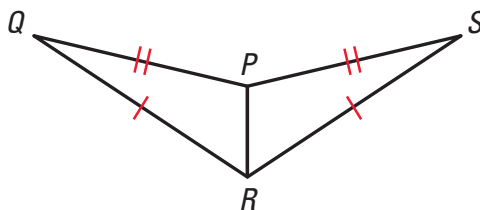


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EXAMPLE 3 *Choosing Which Congruence Postulate to Use*

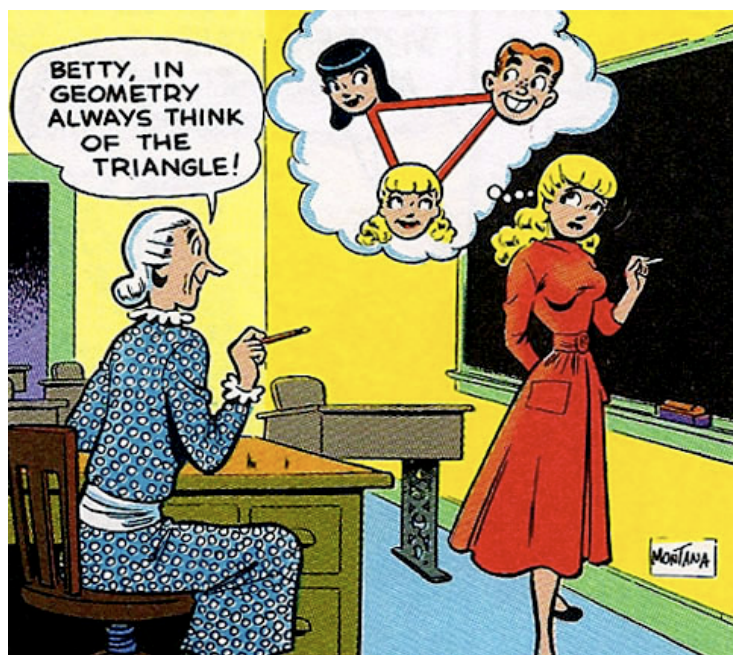
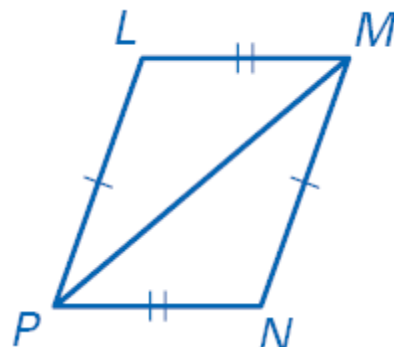
Decide whether enough information is given in the diagram to prove that $\triangle PQR \cong \triangle PSR$. If there is enough information, state the congruence postulate you would use.



SOLUTION

Paragraph Proof The marks on the diagram show that $\overline{PQ} \cong \overline{PS}$ and $\overline{QR} \cong \overline{SR}$. By the Reflexive Property of Congruence, $\overline{PR} \cong \overline{PR}$. Because the sides of $\triangle PQR$ are congruent to the corresponding sides of $\triangle PSR$, you can use the SSS Congruence Postulate to prove that the triangles are congruent.

5. Is there enough information to prove $\triangle LMP \cong \triangle NPM$?



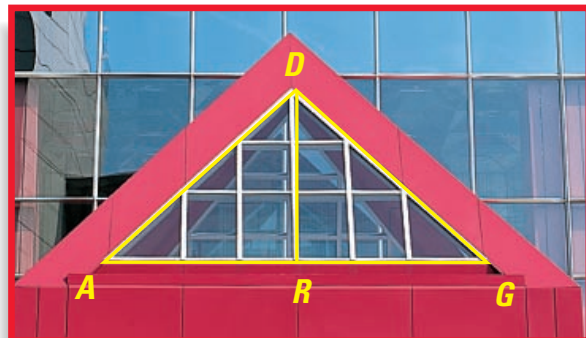
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EXAMPLE 4 Proving Triangles Congruent

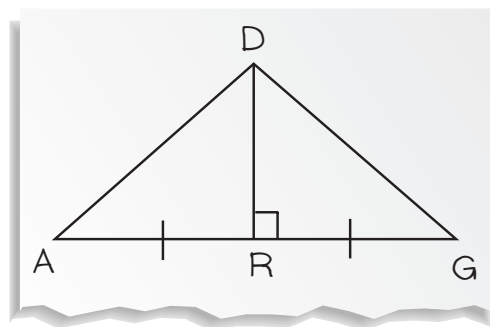


ARCHITECTURE You are designing the window shown in the photo. You want to make $\triangle DRA$ congruent to $\triangle DRG$. You design the window so that $\overline{DR} \perp \overline{AG}$ and $\overline{RA} \cong \overline{RG}$. Can you conclude that $\triangle DRA \cong \triangle DRG$?



SOLUTION

To begin, copy the diagram and label it using the given information. Then write the given information and the statement you need to prove.



Proof

GIVEN ► $\overline{DR} \perp \overline{AG}$,
 $\overline{RA} \cong \overline{RG}$

PROVE ► $\triangle DRA \cong \triangle DRG$

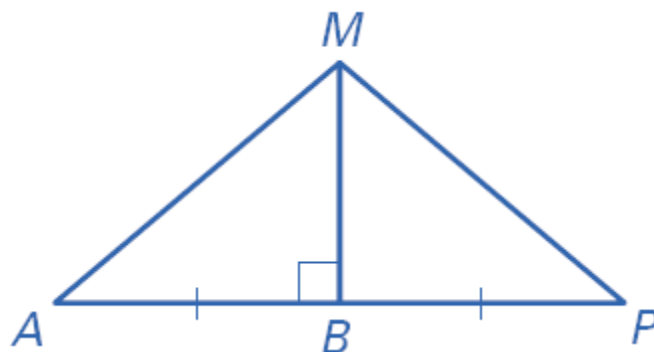
Statements	Reasons
1. $\overline{DR} \perp \overline{AG}$	1. Given
2. $\angle DRA$ and $\angle DRG$ are right angles.	2. If 2 lines are \perp , then they form 4 rt. \angle s.
3. $\angle DRA \cong \angle DRG$	3. Right Angle Congruence Theorem
4. $\overline{RA} \cong \overline{RG}$	4. Given
5. $\overline{DR} \cong \overline{DR}$	5. Reflexive Property of Congruence
6. $\triangle DRA \cong \triangle DRG$	6. SAS Congruence Postulate

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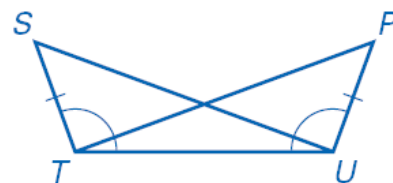
6. Complete the proof.

Given: $\overline{AB} \cong \overline{PB}$
 $\overline{MB} \perp \overline{AP}$

Prove: $\triangle MBA \cong \triangle MBP$



7. Is there enough information to prove that $\triangle STU \cong \triangle PUT$?



EXAMPLE 5

Triangular Frameworks are Rigid



STRUCTURAL SUPPORT To prevent a doorway from collapsing after an earthquake, you can reinforce it. Explain why the doorway with the diagonal brace is more stable, while the one without the brace can collapse.



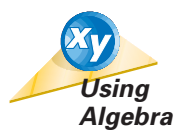
SOLUTION

In the doorway with the diagonal brace, the wood forms triangles whose sides have fixed lengths. The SSS Congruence Postulate guarantees that these triangles are rigid, because a triangle with given side lengths has only one possible size and shape. The doorway without the brace is unstable because there are many possible shapes for a four-sided figure with the given side lengths.

8. A picture frame is assembled with a diagonal brace along the back. Explain why the frame with a brace is more stable than a frame without the brace.

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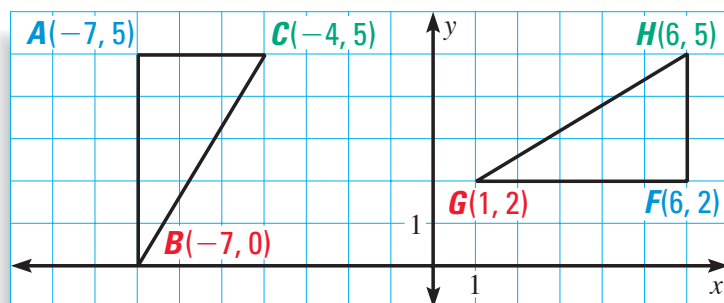
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EXAMPLE 6

Congruent Triangles in a Coordinate Plane

Use the SSS Congruence Postulate to show that $\triangle ABC \cong \triangle FGH$.



SOLUTION

Because $AC = 3$ and $FH = 3$, $\overline{AC} \cong \overline{FH}$. Because $AB = 5$ and $FG = 5$, $\overline{AB} \cong \overline{FG}$. Use the Distance Formula to find the lengths BC and GH .

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$BC = \sqrt{(-4 - (-7))^2 + (5 - 0)^2}$$

$$= \sqrt{3^2 + 5^2}$$

$$= \sqrt{34}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$GH = \sqrt{(6 - 1)^2 + (5 - 2)^2}$$

$$= \sqrt{5^2 + 3^2}$$

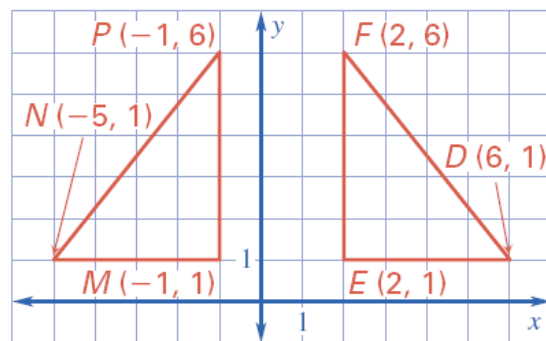
$$= \sqrt{34}$$

- Because $BC = \sqrt{34}$ and $GH = \sqrt{34}$, $\overline{BC} \cong \overline{GH}$. All three pairs of corresponding sides are congruent, so $\triangle ABC \cong \triangle FGH$ by the SSS Congruence Postulate.

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9. Use the SSS postulate to show that

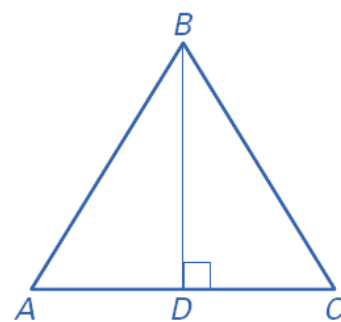
$$\triangle NPM \cong \triangle DFE.$$



10. Explain why the crossed legs of a picnic table give it more support than if the legs go straight down, as in a dinner table.

11. _____ **Multiple Choice.** Which additional congruence is correct to prove $\triangle ABD \cong \triangle CBD$. State the postulate you would use.

- A. $\overline{AB} \cong \overline{BC}$ by SAS
- B. $\overline{AD} \cong \overline{DC}$ by SAS
- C. $\overline{AB} \cong \overline{BC}$ by SSS
- D. $\overline{AD} \cong \overline{DC}$ by SSS



12. Sketch a triangle and label its vertices. Name two sides and the included angle between the sides.

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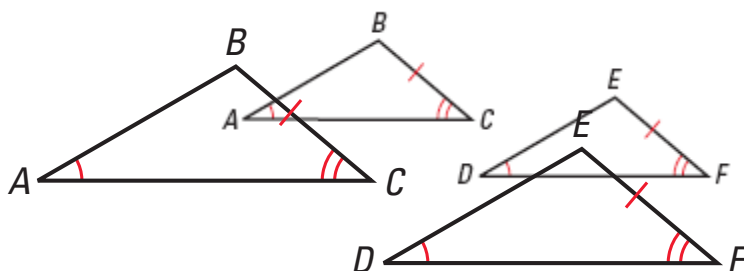
13. Henry believes he can use the information given in the diagram and the SAS Congruence Postulate to prove the two triangles are congruent. Explain Henry's mistake.



A proof of the Angle-Angle-Side (AAS) Congruence Theorem is given below.

GIVEN ► $\angle A \cong \angle D$, $\angle C \cong \angle F$,
 $\overline{BC} \cong \overline{EF}$

PROVE ► $\triangle ABC \cong \triangle DEF$



Paragraph Proof You are given that two angles of $\triangle ABC$ are congruent to two angles of $\triangle DEF$. By the Third Angles Theorem, the third angles are also congruent. That is, $\angle B \cong \angle E$. Notice that \overline{BC} is the side included between $\angle B$ and $\angle C$, and \overline{EF} is the side included between $\angle E$ and $\angle F$. You can apply the ASA Congruence Postulate to conclude that $\triangle ABC \cong \triangle DEF$.

Ways to Prove Triangles Congruent

Ways that you cannot use to prove triangles congruent.

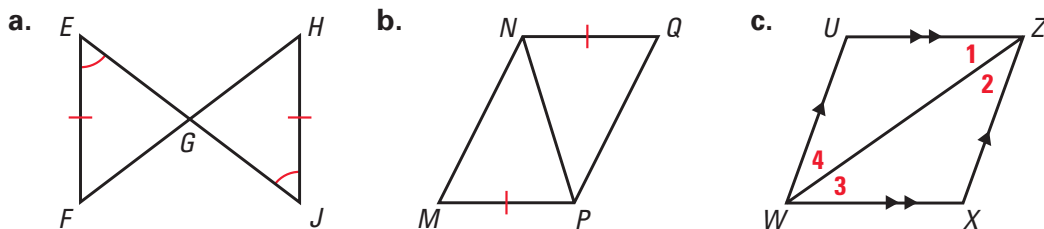
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EXAMPLE 1 Developing Proof

Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem you would use. Explain your reasoning.

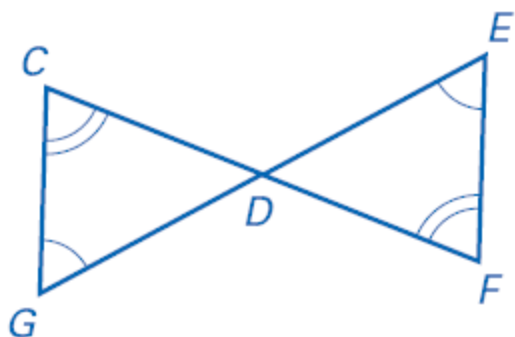


SOLUTION

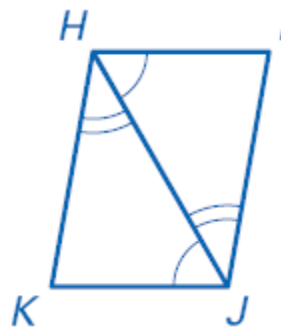
- In addition to the angles and segments that are marked, $\angle EGF \cong \angle JGH$ by the Vertical Angles Theorem. Two pairs of corresponding angles and one pair of corresponding sides are congruent. You can use the AAS Congruence Theorem to prove that $\triangle EFG \cong \triangle JHG$.
- In addition to the congruent segments that are marked, $\overline{NP} \cong \overline{NP}$. Two pairs of corresponding sides are congruent. This is not enough information to prove that the triangles are congruent.
- The two pairs of parallel sides can be used to show $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$. Because the included side \overline{WZ} is congruent to itself, $\triangle WUZ \cong \triangle ZXW$ by the ASA Congruence Postulate.

Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem.

2.



3.



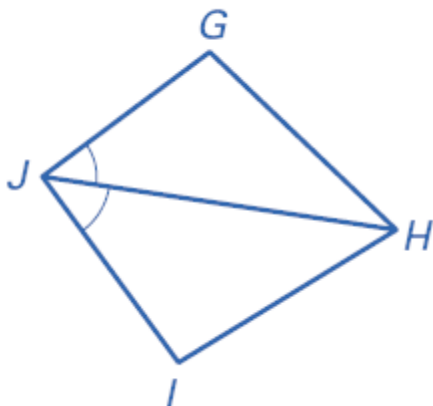
STUDENT HELP

Study Tip

In addition to the information that is marked on a diagram, you need to consider other pairs of angles or sides that may be congruent. For instance, look for vertical angles or a side that is shared by two triangles.

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4. Is it possible to prove that the triangles are congruent? If so, state the postulate or theorem.

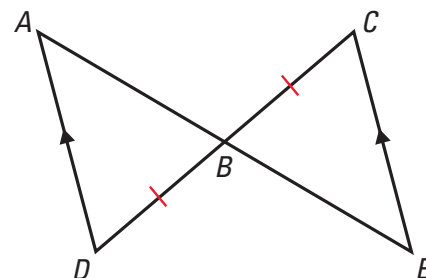


EXAMPLE 2 **Proving Triangles are Congruent**

GIVEN ▶ $\overline{AD} \parallel \overline{EC}$, $\overline{BD} \cong \overline{BC}$

PROVE ▶ $\triangle ABD \cong \triangle EBC$

Plan for Proof Notice that $\angle ABD$ and $\angle EBC$ are congruent. You are given that $\overline{BD} \cong \overline{BC}$. Use the fact that $\overline{AD} \parallel \overline{EC}$ to identify a pair of congruent angles.



Statements	Reasons
1. $\overline{BD} \cong \overline{BC}$	1. Given
2. $\overline{AD} \parallel \overline{EC}$	2. Given
3. $\angle D \cong \angle C$	3. Alternate Interior Angles Theorem
4. $\angle ABD \cong \angle EBC$	4. Vertical Angles Theorem
5. $\triangle ABD \cong \triangle EBC$	5. ASA Congruence Postulate

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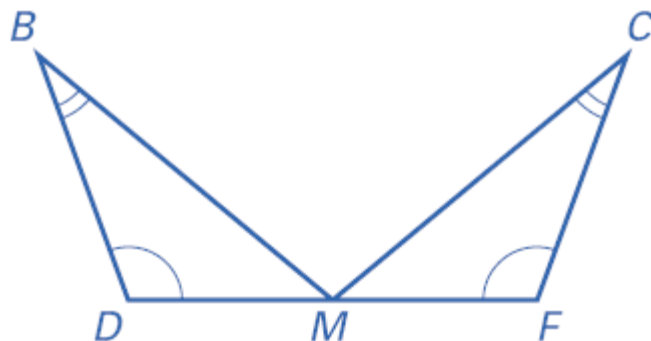
5. Example.

$$\angle B \cong \angle C$$

Given: $\angle D \cong \angle F$

M is the midpoint of \overline{DF} .

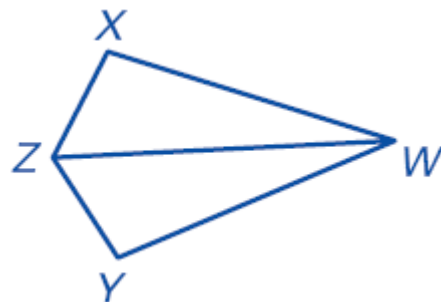
Prove: $\triangle BDM \cong \triangle CFM$



6.

Given: \overline{WZ} bisects $\angle XZY$ & $\angle XWY$.

Prove: $\triangle WZX \cong \triangle WZY$



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EXAMPLE 3 *Using Properties of Congruent Triangles*

FOCUS ON APPLICATIONS



METEORITES

When a *meteoroid* (a piece of rocky or metallic matter from space) enters Earth's atmosphere, it heats up, leaving a trail of burning gases called a *meteor*. Meteoroid fragments that reach Earth without burning up are called *meteorites*.

METEORITES On December 9, 1997, an extremely bright meteor lit up the sky above Greenland. Scientists attempted to find meteorite fragments by collecting data from eyewitnesses who had seen the meteor pass through the sky. As shown, the scientists were able to describe sightlines from observers in different towns. One sightline was from observers in Paamiut (Town P) and another was from observers in Narsarsuaq (Town N).

Assuming the sightlines were accurate, did the scientists have enough information to locate any meteorite fragments? Explain.

SOLUTION

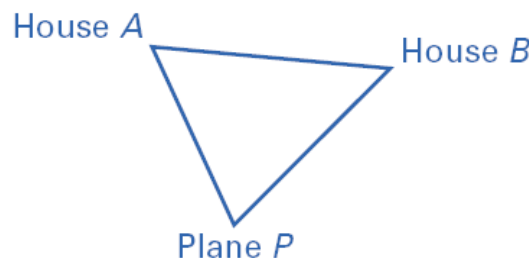
Think of Town P and Town N as two vertices of a triangle. The meteorite's position M is the other vertex. The scientists knew $m\angle P$ and $m\angle N$. They also knew the length of the included side \overline{PN} .

From the ASA Congruence Postulate, the scientists could conclude that any two triangles with these measurements are congruent. In other words, there is only one triangle with the given measurements and location.



- Assuming the sightlines were accurate, the scientists did have enough information to locate the meteorite fragments.

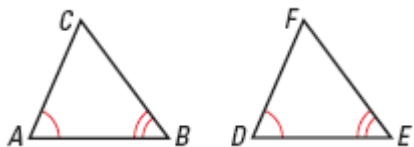
7. When searching for a missing airplane, searchers used observations from people in two different areas of a city. As shown, the observers were able to describe sight lines from observers in different houses. One sightline was from observers in House A and the other sightline was from observers in House B. Assuming the sightlines are accurate, did the searchers have enough information to locate the airplane?



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State the third congruence that must be given to prove that using the indicated postulate or theorem.

8. ASA Congruence



9. AAS Theorem

